Nonextensive models for earthquakes

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We have revisited the fragment-asperity interaction model recently introduced by Sotolongo-Costa and Posadas [Phy. Rev. Lett. **92**, 048501 (2004)] by considering a different definition for mean values in the context of Tsallis nonextensive statistics and introducing a scale between the earthquake energy and the size of fragment $\epsilon \propto r^3$. The energy-distribution function (EDF) deduced in our approach is considerably different from the one obtained in the above reference. We have also tested the viability of this EDF with data from two different catalogs (in three different areas), namely, the NEIC and the *Bulletin Seismic of the Revista Brasileira de Geofísica*. Although both approaches provide very similar values for the nonextensive parameter q, other physical quantities, e.g., energy density, differ considerably by several orders of magnitude.

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I. INTRODUCTION

Over the last two decades a great deal of attention has been paid to the so-called nonextensive Tsallis entropy both from theoretical and observational viewpoints. This particular nonextensive formulation [1,2] seems to present a consistent theoretical tool to investigate complex systems in their nonequilibrium stationary states, systems with multifractal and self-similar structures, systems dominated by long-range interactions, and anamolous diffusion phenomena among others. Some recent applications of the Tsallis entropy $S_{q\neq 1}$ to a number of complex scenarios is now providing a more definite picture of the kind of physical problems to which this q formalism can in fact be applied.

In this regard, systems of interest in geophysics have also been studied in light of this nonextensive formalism. In this particular context an investigation was done by Abe [3] who showed that the statistical properties of the three-dimensional distance between successive earthquakes follows a q-exponential function with the nonextensive parameter lying in the interval [0, 1] [4]. Since then, other geophysical analyses have been performed such as, for instance, the statistics of the calm time, which indicates a scale-free nature for earthquake phenomena and corresponds to a q-exponential distribution with $q>1$ [5], models for temperature distributions, and radon emission of volcanos [6]. More recently, a very interesting model for earthquake dynamics related to the Tsallis nonentensive framework has been proposed by the Sotolongo-Costa and Posadas (SCP) model [7]. Such a model consists basically of two rough profiles interacting via fragments filling the gap between them where the fragments are produced by local breakage of the local plates. By using the nonextensive formalism the authors of Ref. [7] not only showed the influence of the size distribution of fragments on the energy distribution of earthquakes but also deduced an energy-distribution function (EDF) that gives the well-known Gutenberg-Richter law [8] as a particular case.

However, in dealing with this nonextensive framework, particular attention must be paid to the possible definitions for mean values, which play a fundamental role within the domain of these nonextensive statistics [9]. In this regard, recent studies of the properties of the relative entropy and the Shore-Johnson theorem for a consistent minimum cross-entropy principle, revealed the necessity of the so-called q-expectation value in studies involving these nonextensive statistical mechanics (see Ref. [10] for details). Thus, by introducing this q definition of mean value we reanalyzed the fragment-asperity interaction model of Sotolongo-Costa and Posadas [7]. Moreover, a scale law between the released relative energy ϵ and the three-dimensional size of fragments has also been introduced. By using the standard method of entropy maximization we also deduced an energy distribution function that differs considerably from the one obtained in Ref. [7]. In order to test the viability of our appoach we used data taken from two seismic catalogs, namely, the NEIC and the *Bulletin Seismic of the Revista Brasileira de Geofísica*. It is shown that although both approaches provide very similar values for the nonextensive parameter q, other physical quantities, e.g., energy density, differ by several orders of magnitude.

This paper is organized as follows. In Sec. II the standard formalism of nonextensive statistical mechanics is reexam-

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ined as well as the theorical basis of the SCP model. In Sec. III an EDF is calculated analytically through the extremization of the Tsallis entropy under the constrains of the q-expectation value and the normalization condition. In Sec. IV we test this EDF with data from two different catalogs and estimate the best-fit values for the nonextensive parameter q and the proportionality constant between the released relative energy ϵ and the volume of the fragments r^3, i.e., the energy density a. We summarize our main results in Sec. V.

II. NONEXTENSIVE FRAMEWORK AND SCP MODEL

In this section we recall the nonextensive theoretical basis of the SCP model. As is widely known, the Tsallis statistics generalize the Botzmann-Gibbs statistics as far as the concept of entropy is concerned. Such formalism is based on the parametric class of entropies given by

$$S_{q\neq 1} = -k_B \int p^q(\sigma)\ln_q p(\sigma)d\sigma, \tag{1}$$

where k_B is the Boltzmann constant. In the SCP model $p(\sigma)$ stands for the probability of finding a fragment of relative surface σ (which is defined as a characteristic surface of the system), q is the nonextensive parameter, and the q-logarithmic function above is defined by

$$\ln_q p = (1-q)^{-1}(p^{1-q}-1) \quad (p > 0), \tag{2}$$

which recovers the standard Boltzmann-Gibbs entropy $S_1 = -k_B \int p \ln p d^3 p$ in the limit $q \to 1$. It is worth mentioning that most of the experimental evidence supporting the Tsallis proposal are related to the power-law distribution associated with the $S_{q\neq 1}$ descripition of the classical N-body problem [11].

The SCP model is a simple approach for earthquake dynamics revealing a very interesting application of the Tsallis framework. Indeed, the fundamental idea is based on the fact that the space between faults is filled with the residue of the breakage of the tectonic plates. In this regard, the authors studied the influence of the size distribution of fragments on the energy distribution of earthquakes. The theoretical motivation follows from the fragmentation phenomena [12] in the context of the geophysical systems. In this latter work, Englaman *et al.* showed that the standard Botzmann-Gibbs formalism, although useful, cannot account for an important feature of fragmentation process, i.e., the presence of scaling in the size distribution of fragments, which is one of the main ingredients of the SCP approach. Thus, a nonextensive formalism is not only justified in the SCP model but it is also necessary since the process of violent fractioning is very probably a nonextensive phenomenon leading to long-range interactions between the parts of the object being fragmented (see, e.g., [7,13]). In reality, such an influence was emphasized earlier in other investigations [14]. In general, the SCP model follows similar arguments to those presented in Ref. [15] being, however, a more realistic seismic model than the one proposed in Ref. [16]. In particular, the theoretical ingredients read as follows:

(i) The mechanism of relative displacement of fault plates is the main cause of earthquakes.

(ii) The surfaces of the tectonic plates are irregular and the fragments filling the space between them are very diverse and have irregular shapes.

(iii) The mechanism of triggering earthquakes is established through the combination of the irregularities of the fault planes and the distribution of fragments between them.

(iv) The fragment-distribution function, and consequently the EDF, emerges naturally from a nonextensive framework.

From the above arguments, the EDF deduced in Ref. [7] is given by

$$\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right) \times \log[1 + a(q-1)(2-q)^{(1-q)/(q-2)} \times 10^{2m}]. \tag{3}$$

According to Ref. [7], the above expression describes the energy distribution in all detectable range of magnitudes very well, unlike the empirical formula of Gutenberg-Richter [8].

III. CURRENT APPROACH

Now let us discuss the standard method of maximization of the Tsallis entropy. In this section and hereafter the Boltzmann constant is set equal to unity for the sake of simplicity. Thus, the functional entropy to be maximized is

$$\delta S_q^* = \delta\left(S_q + \alpha\int_0^\infty p(\sigma)d\sigma - \beta\sigma_q\right) = 0, \tag{4}$$

where α and β are the Lagrange multipliers. The constraints used above are the normalization of the distribution

$$\int_0^\infty p(\sigma)d\sigma = 1 \tag{5}$$

and the q-expectation value

$$\sigma_q = \langle\sigma\rangle_q = \int_0^\infty \sigma P_q(\sigma)d\sigma \tag{6}$$

with the escort distribution [17] given by

$$P_q = \frac{p^q(\sigma)}{\int_0^\infty p^q(\sigma)d\sigma}. \tag{7}$$

By considering the same physical arguments of Ref. [7], we derive, after some algebra, the following expresion for the fragment size distribution function:

$$p(\sigma) = \left[1 - \frac{(1-q)}{(2-q)}(\sigma - \sigma_q)\right]^{1/(1-q)}, \tag{8}$$

which corresponds to the area distribution for the fragments of the fault plates. Here, however, and differently from Ref. [7], which assumes that $\varepsilon \sim r$, we use an energy scale of $\varepsilon \sim r^3$. Thus, the proportionality between the released relative energy ϵ and r^3 (r is the size of fragments) is now given by $\sigma - \sigma_q = (\varepsilon/a)^{2/3}$, where σ scales with r^2 and a (the proportionality constant between ε and r^3) and has the dimension of volumetric energy density. In particular, this scale is in accordance with the standard theory of seismic rupture, the well-known seismic moment scaling with rupture length (see, for example, Ref. [18]).

The EDF of earthquakes is obtained by changing variables from $\sigma - \sigma_q$ to $(\varepsilon/a)^{2/3}$. From Eq. (8) it is straightforward to show that

$$p(\varepsilon)d\varepsilon = \frac{C\varepsilon^{-1/3}d\varepsilon}{[1 + C'\varepsilon^{2/3}]^{1/(q-1)}}, \tag{9}$$

which has also a power-law form with C and C' given by

$$C = \frac{2}{3a^{2/3}} \quad \text{and } C' = -\frac{(1-q)}{(2-q)a^{2/3}}. \tag{10}$$

In the above expression the energy probability is written as $p(\varepsilon) = n(\varepsilon)/N$, where $n(\varepsilon)$ corresponds to the number of earthquakes with energy ε and N is the total number of earthquakes.

IV. TESTING THE EDF WITH THE CUMULATIVE NUMBER OF EARTHQUAKES

In order to test the viability of the EDF derived above [Eq. (9)] we introduce the cumulative number of earthquakes given by integral [7]

$$\frac{N_{\epsilon>}}{N} = \int_{\varepsilon}^{\infty} p(\varepsilon)d\varepsilon, \tag{11}$$

where $N_{\varepsilon>}$ is the number of earthquakes with energy larger than ε. Now, substituting Eq. (9) into Eq. (11), and considering $m = \frac{1}{3}\ln \varepsilon$ (m stands for magnitude) it is possible to calculate the above expression. In reality, note that depending on the value of q the limits of integral (11) presents a cutoff on the maximum value allowed for energy ϵ, which is given by $\epsilon_{\max} = \sqrt{a^{2/3}(2-q)/(1-q)}$ for the intervals $q < 1$ and $q > 2$, while for $1 < q < 2$ the cutoff is absent in the distribution. Note also that in the limit $q \to 1$, $\epsilon_{\max} \to \infty$, and $p(\varepsilon)$ goes to the exponential function. As a matter of fact, the calculation of integral (11) for $q \neq 1$ leads to the general expression

$$\log(N_{>m}) = \log N + \left(\frac{2-q}{1-q}\right)\log\left[1 - \left(\frac{1-q}{2-q}\right)\left(\frac{10^{2m}}{a^{2/3}}\right)\right], \tag{12}$$

which, similarly to the modified Gutenberg-Ricther law (see, e.g., Ref. [19] for more details), appropriately describes the energy distribution in a wider detectable range of magnitudes.

Figure 1 shows the relative cumulative number of earthquakes ($G_{m>} = N_{m>}/N$) as a function of the magnitude m. The data points, corresponding to earthquakes events lying in the interval $3 < m < 8$, were taken from two different catalogs, namely, the *Bulletin Seismic of the Revista Brasileira de Geofísica* [Fig. 1(a)] and the NEIC [Figs. 1(b) and 1(c)]. Figures 1(a), 1(b), and 1(c) show the results of our analysis for the Samambaia fault, Brazil (100 events), the New Madrid fault, USA (173 events), and the Anatolian fault, Turkey (8980 events), respectively. We note that similarly to the original version of the SCP model, our approach, as represented by Eqs. (8)–(12), provide a very good fit to the experimental data of the two catalogs considered here. It is worth emphasizing, however, that the energy densities differ by several orders of magnitude from our model to the original SCP model. Therefore, we expect that other independent estimates of the parameter a may indicate which approach is more realistic physically. The estimates of the parameters q and a obtained in this paper and in Ref. [7] are summarized in Table I.

V. CONCLUSION

In Ref. [10] we discussed what seems to be the correct definition for the expectation values within the Tsallis nonextensive statistical mechanics. Based on the properties of the generalized relative entropies and the Shore-Johnson theorem, it was shown that the expectation value of any physical quantity in this extended framework converges to the normalized q-expectation value instead of to the ordinary definition.

In this paper, by considering this necessity of q-expectation values in the Tsallis nonextensive framework, we have revisited the fragment-asperity interaction model for earthquakes as introduced in Ref. [7]. An energy-distribution function has been calculated, which allowed us to determine the relative cumulative number of earthquakes as a function of the magnitude. Additionally, a scale law between the re-

TABLE I. Limits to q and a.

Fault	Reference	q	a
California, USA	[7]	1.65	5.73×10^{-6}
Iberian Penisula, Spain	[7]	1.64	3.37×10^{-6}
Andalucía, Spain	[7]	1.60	3.0×10^{-6}
Samambaia, Brazil	This paper	1.60	1.3×10^{10}
New Madrid, USA	This paper	1.63	1.2×10^{10}
Anatolian, Turkey	This paper	1.71	2.8×10^{10}

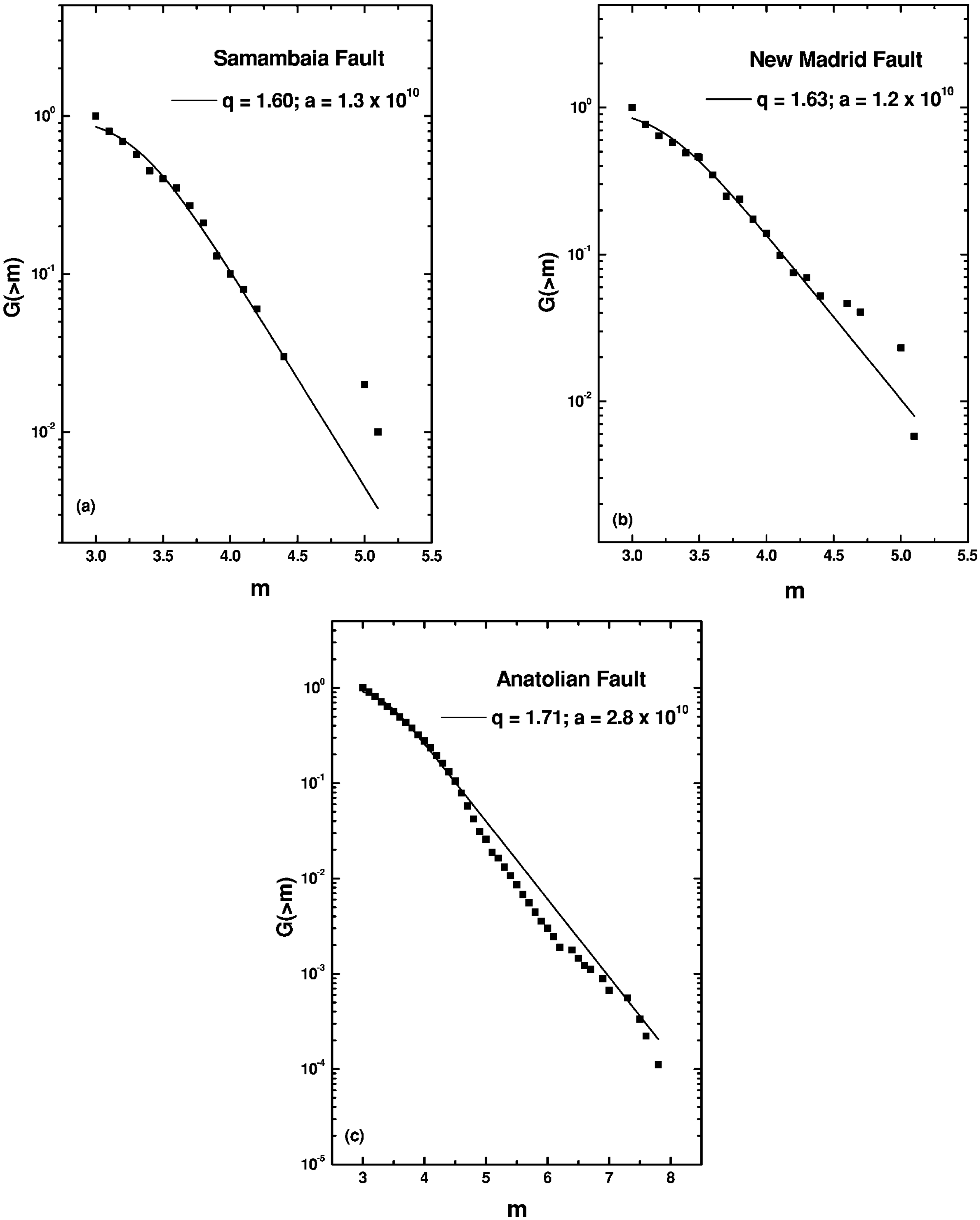

FIG. 1. The relative cumulative number of earthquakes [Eq. (12)] as a function of the magnitude m. In all panels the data points correspond to earthquakes lying in the interval $3<m<8$. (a) Samambaia fault, Brazil: 100 events from *Bulletin Seismic of the Revista Brasileira de Geofísica*. (b) New Madrid fault, USA: 173 data points taken from the NEIC catalog. (c) Anatolian fault, Turkey: 8980 events from the NEIC catalog. The best-fit values for the parameters q and a are shown in the respective panels. A summary of this analysis is also shown in Table I.

leased relative energy ϵ and the volume of fragments r^3 has also been introduced, i.e., in agreement with the so-called seismic moment scaling with rupture length. As discussed earlier, although our analysis and the one presented in Ref. [7] provide very similar values for the nonextensive parameter q, the other physical quantities, e.g., energy density, differ by several orders of magnitude. It would be interesting, therefore, if we could have experimental estimates for these quantities in order to compare the predictions of the models. Finally, it is worth mentioning that the estimates for the nonextensive parameter from the two catalogs considered here (Fig. 1) are consistent with the upper limit $q < 2$ obtained from several independent studies involving the Tsallis nonextensive framework [20].

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